Student Number:	
Class:	

TRIAL HIGHER SCHOOL CERTIFICATE EXAMINATION 2015

MATHEMATICS

General Instructions:

- Reading Time: 5 minutes.
- Working Time: 3 hours.
- Write in black pen.
- Board approved calculators & templates may be used
- A Standard Integral Sheet is provided.
- In Question 11 - 16, show all relevant mathematical reasoning and/or calculations.
- Marks may not be awarded for careless or badly arranged working.

Total Marks 100

Section I: 10 marks

- Attempt Question 1 – 10.
- Answer on the Multiple Choice answer sheet provided.
- Allow about 15 minutes for this section.

Section II: 90 Marks

- Attempt Question 11 - 16
- Answer on lined paper provided. Start a new page for each new question.
- Allow about 2 hours & 45 minutes for this section.

The answers to all questions are to be returned in separate *stapled* bundles clearly labelled Question 11, Question 12, etc. Each question must show your Candidate Number.

Multiple Choice Questions

Choose the best answer for each of the following questions.

1. For what values of k does the equation $x^2 - 6x - 3k = 0$ have real roots?
   A $k \le -3$ B $k \ge -3$ C $k \le 3$ D $k \ge 3$

2. Two ordinary dice are rolled. The "score " is the sum of the numbers on the top faces. What is the probability that the scores is 9?
   A $\frac{1}{9}$ B $\frac{1}{4}$ C $\frac{1}{3}$ D $\frac{3}{4}$

3. Express $\frac{\sqrt{5}}{1+\sqrt{2}}$ in the form of $\sqrt{a} - \sqrt{b}$ where a and b are rational numbers.

   A $\sqrt{10} - \sqrt{5}$ B $\sqrt{5} - \sqrt{10}$ C $(\sqrt{10} - \sqrt{5})/3$ D $(\sqrt{5} - \sqrt{10})/3$

4. Find the derivative of $\cos^2 3x$ with respect to x.
   A $-2\sin 3x \cos 3x$ B $-6\sin 3x \cos 3x$
   C $2\sin 3x \cos 3x$ D $2\sin 3x \cos 3x$

5. Evaluate $\int_0^1 (e^{-3x} - 1)dx$.

   A $-(\frac{e^{-3}}{3}+1)$ B $-\frac{e^{-3}}{3}+\frac{2}{3}$ C $-(\frac{e^{-3}}{3}+\frac{2}{3})$ D $\frac{1}{3}(e^{-3}-1)$

6. What are the domain and range of $f(x) = \sqrt{4 - x^2}$?
   A Domain: $-2 \le x \le 2$ Range: $0 \le y \le 2$
   B Domain: $-2 \le x \le 2$ Range: $-2 \le y \le 2$
   C Domain: $0 \le x \le 2$ Range: $-4 \le y \le 4$
   D Domain: $0 \le x \le 2$ Range: $0 \le y \le 4$

7. Daniel planted a bed of gardenias in rows on his commercial property. Each row had to be fertilised before planting.

   There were 13 gardenia plants in the first row, 19 gardenia plants in the second row, and so on. Each succeeding row had 6 more gardenia plants than the row before it.

   If Daniel wanted to plant 1453 gardenias, how many rows will he need to fertilise?

   A 20.28 B 20.40 C 23.61 D 23.74

8. A particle moves so that its velocity function at time t seconds, is given by : $v = 2e^{-t}(1-t)$.
Find the time when the acceleration is zero.

A $t = 0$ B $t = 1$ C t=2 D t=3

9. Find the perimeter (P)of the sector of a circle with a radius of 20cm and an angle 36° subtended at the centre.

A P=$0.5 \times 400 \times \left(\frac{\pi}{5} - \sin\frac{\pi}{5}\right)$cm

B P= $\left(0.5 \times 400 \times \frac{\pi}{5}\right)$ cm

C P= $(40 + \frac{\pi}{5})$ cm

D P=$(40 + 4\pi)$cm

10. Find the values of x for which the geometric series $2 + 4x + 8x^2 + \dots$ has a limting sum.

A $x < \frac{1}{2}$ B $x \geq \frac{1}{2}$ C $|x| \leq \frac{1}{2}$ D $|x| < \frac{1}{2}$

Question 11 **Marks**

a. Evaluate $\lim_{x \to 2} \frac{x^3 - 8}{x - 2}$. 2

b. Find $\int \left(\sqrt{5x - 1}\right) dx$. 1

c. Evaluate $\int_1^3 \frac{3x}{x^2 + 4} dx$. 2

d. Differentiate $y = \frac{\sin x}{1 + \cos x}$ and simplify. 2

e. (i) Differentiate $x \ln x$. 1

(ii) Hence find $\int \ln x \, dx$. 2

f. If a, b and c are consecutive terms of a geometric sequence, show that ln a, ln b and ln c are consecutive terms of an arithmetic sequence. 2

g. A parabola in the coordinate plane is represented by the equation $x^2 - 10x - 16y - 7 = 0$.

(i) Find the coordinates of the vertex. 2

(ii) Find the focal length. 1

Question 12 (Start a new page)

a. A Geiger counter is taken into a region after a nuclear accident and gives a reading of 10 000 units. One year later, the same Geiger counter gives a reading of 9000 units. It is known that the reading is given by the formula $T = T_0 e^{-kt}$, where T_0 and k are constants and t is the time, measured in years.

(i) Evaluate the exact values of T_0 and k. 2

(ii) It is known that the region will become safe after the reading reaches 40 units. After how many years will the region become safe? 2

(iii) Sketch the graph of $T = T_0 e^{-kt}$. 1

b. (i) On a Cartesian plane, plot the points A, B and C which are (-4,3), (0,5) and (9,2) respectively. 1

(ii) Find the length of the interval BC. 1

(iii) Show that the equation of the line l, drawn through A and parallel to BC is $x + 3y - 5 = 0$. 2

(iv) Find the co-ordinates of D, the point where the line l meets the x-axis. 1

(v) Prove that $ABCD$ is a parallelogram. 2

(vi) Find the perpendicular distance from the point B to the line l. 2

(vii) Hence or otherwise find the area of the parallelogram $ABCD$. 1

Question 13 (Start a new page)

a. A particle is moving on the x-axis. It starts from the origin, and at the time t seconds, its velocity v m/s is given by $v = 1 - 2\sin t$.
Let $t = t_1$, $t = t_2$ be the first two times when the particle comes to rest.

(i) Find t_1 and t_2. 2

(ii) Sketch the velocity function for $0 \le t \le 2\pi$. 2

(iii) Find the acceleration at t_1 and t_2. 2

(iv) Find the displacement function. 2

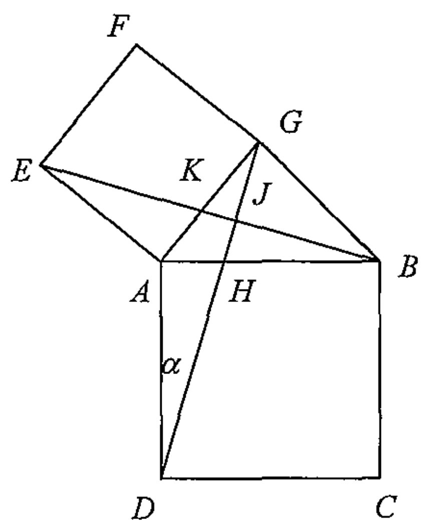
(v) Hence, or otherwise, find the exact distance travelled between t_1 and t_2. 2

b. α, β are the roots of the quadratic equation $2x^2 - (4k+1) + 2k^2 - 1 = 0$. 2
If $\alpha = -\beta$, find the value of k.

c. Given that $f(x) = 4x - 3$ is the gradient function of a curve and the line $y = 5x - 7$ is tangent to the curve. 3
Find the equation of the curve.

Question 14 (Start a new page)

a.



Not to scale

Two squares $ABCD$ and $AEFG$ are drawn above. AG and EB intersect at K and DG and AB intersect at H. Let $\angle ADG = \alpha$.

Copy the diagram into your writing booklet.

(i) Prove that $\Delta ADG \equiv \Delta ABE$. 2

(ii) Prove that $EB \perp DG$. 3

Question 14 (continued)

b. A bag contains 2 red balls, one black ball, and one white ball. Ming selects one ball from the bag and keeps it hidden. He then selects a second ball, and also keeping it hidden.

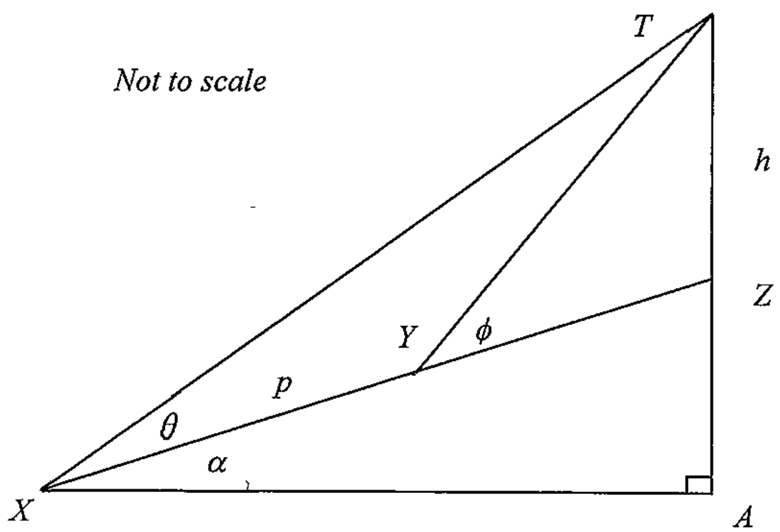
(i) Draw a tree diagram to show all the possible outcomes. 1

(ii) Find the probability that both the selected balls are red. 1

(iii) Find the probability that at least one of the selected balls is red. 1

(iv) Ming drops one of the selected balls and we can see that it is red. What is the probability that the ball that is still hidden is also red? 1

c.



In the diagram above, TXA is a right-angled triangle.
$XY = p$, $TZ = h$, $\angle TYZ = \phi$, $\angle ZXA = \alpha$, $\angle TXY = \theta$.

Copy the diagram into your writing booklet. 2

(i) Consider ΔXYT in the above diagram, show that $TY = \dfrac{p\sin\theta}{\sin(\phi - \theta)}$.

(ii) Show that $\angle YZT = \dfrac{\pi}{2} + \alpha$. 1

(iii) Hence, use part (i) and (ii) to show that $h = \dfrac{p\sin\theta\sin\phi}{\sin(\phi-\theta)\cos\alpha}$. 3

Question 15 (Start a new page)

a. Graph the solution of $4x \le 15 \le -9x$ on a number line. 3

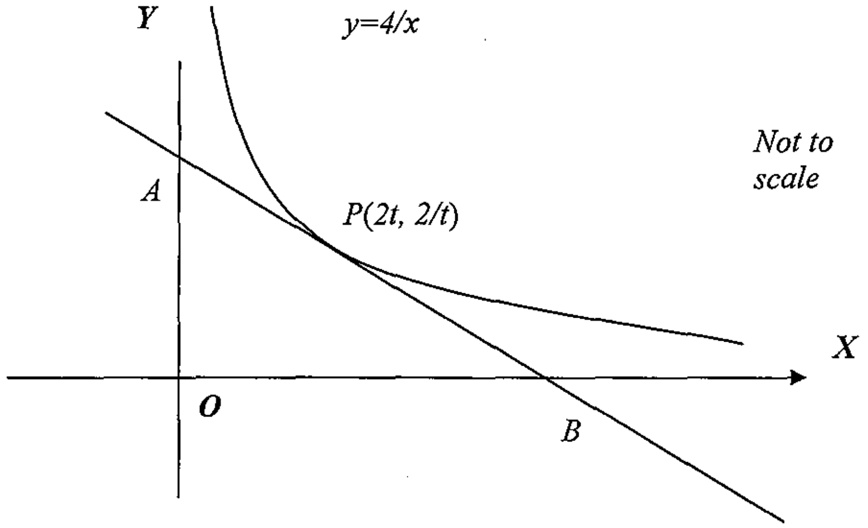
b. (i) Find the area bounded by the curve $y = \tan 2x$, $0 \le x \le \dfrac{\pi}{6}$ and the x-axis 3

(ii) The region bounded by the curve $y = \tan 2x$, $0 \le x \le \dfrac{\pi}{6}$ and the x-axis is rotated about the x-axis and form a solid. 3

Find the volume of this solid using two applications of Simpson's Rule.

c. In the diagram below, $P(2t, 2/t)$ is a variable point on the branch of the hyperbola $y=4/x$ in the first quadrant.

The tangent at P meets the y-axis at A and the x-axis at B.



(i) Show that the equation of the tangent at P is $t^2y = 4t - x$. 2

(ii) Let the square of the length of AB, ie AB^2, be denoted by v. 4
Find the value of t for which v is a minimum.

Question 16 (Start a new page)

a. If $\log_5 8 = a$, prove that $\log_{10} 2 = \dfrac{a}{a+3}$. 3

b. (i) Justify the graph of $f(x) = x - \dfrac{1}{x^2}$ is always concave down. 2

(ii) Sketch the graph of $f(x) = x - \dfrac{1}{x^2}$, showing all intercept(s) and stationary point(s). 3

c. When Robby is 3 months old, his parents decide to make a regular deposit of \$500 every 3 months, starting with first one when Robby is 3 months old in an account that earns interest of 8%p.a., the interest being paid every 3 months.

(i) Show that the day after Robby's 1st birthday (after payment is made), the value of the account is given by \$ 2060.80. 2

(ii) How much money will be in the account the day when Robby turns 15 after the payment is made? 2

(iii) No more payments are made into the account after Robby turns 15 and no withdrawals are made. 1
Find the amount in the account on Robby's 16th birthday.

(iv) Robby decides that he will withdraw a regular amount of money from this account each birthday, starting with his 16th birthday. He cannot decide whether he should withdraw \$4000 or \$5000 each birthday. 2
By considering the result of part (iii), comment on what will happen in each case.

END of PAPER

1) $\Delta \geq 0$

$36 - 4(-3k) \geq 0$

$36 + 12k \geq 0$

$k \geq -3$ (B)

2) $(3,6), (6,3)\ (5,4), (4,5)$

$\frac{4}{36} = \frac{1}{9}$ (A)

3) $\frac{\sqrt{5}}{1+\sqrt{2}} \times \frac{1-\sqrt{2}}{1-\sqrt{2}} = \frac{\sqrt{5}-\sqrt{10}}{-1} = \sqrt{10} - \sqrt{2}$

(A)

4) $2\cos 3x(-\sin 3x) \times 3$

$= -6\cos 3x \sin 3x$

(B)

5) $\left.\frac{e^{-3x}}{-3} - x\right]_0^1 = \frac{e^{-3}}{-3} - 1 - \left(\frac{1}{-3} - 0\right)$

$= \frac{-e^{-3}}{3} - 1 + \frac{1}{3}$

$= -\frac{1}{3}\left[2 + e^{-3}\right]$ (C)

6) (A)

7) $a = 13 \quad d = 6$

$1453 = \left[\frac{2a + (n-1)d}{2}\right] n$

$2906 = 26n + (n-1)6n$

$0 = 6n^2 + 20n - 2906$

$0 = 3n^2 + 10n - 1453$

$n = \frac{-10 \pm \sqrt{100 - 4(3)(-1453)}}{6}$

$n = \left(-10 \pm \sqrt{17536}\right) \div 6$

$n = 20.40$ (B)

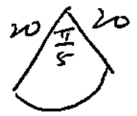
8) $v = 2e^{-t} - 2e^{-t} \cdot t$

$\ddot{x} = -2e^{-t} + (-1+t)2e^{-t}$

$\ddot{x} = 2e^{-t}(-2+t) = 0$

when $t = 2$ (C)

9) $P = 40 + 20 \cdot \frac{\pi}{5}$



$36 = \frac{\pi}{5}$

$P = 40 + 4\pi$

(D)

10) $\frac{1}{|2x|} < 1$ (D)

$|x| < \frac{1}{2}$

2U : 2015 TRIAL MATHEMATICS: Question 11 JRAHS

Suggested Solutions	Marks	Marker's Comments
a) $\lim\limits_{x\to 2} \dfrac{x^3-8}{x-2} = \lim\limits_{x\to 2} \dfrac{\cancel{(x-2)}(x^2+2x+4)}{\cancel{(x-2)}}$	1	correct factorisation
$= \lim\limits_{x\to 2} (x^2+2x+4)$		
$= 12$	1	evaluation of limit.
b) $\int \sqrt{5x-1}\,dx = \dfrac{(5x-1)^{3/2}}{3/2} \cdot \dfrac{1}{5} + C$	1	correct integration
$= \dfrac{2}{15}(5x-1)^{3/2} + C$		
c) $\displaystyle\int_1^3 \frac{3x}{x^2+4}\,dx = \frac{3}{2}\int_1^3 \frac{2x}{x^2+4}\,dx$		
$= \dfrac{3}{2} \ln(x^2+4)\Big\vert_1^3$	1	correct integration
$= \dfrac{3}{2}(\ln 13 - \ln 5)$		
$= \dfrac{3}{2} \ln \dfrac{13}{5}$	1	correct evaluation of integral

2u : 2015 TRIAL **MATHEMATICS: Question...11...** JRAHS

Suggested Solutions	Marks	Marker's Comments
d) $y = \dfrac{\sin x}{1+\cos x}$		
$\dfrac{dy}{dx} = \dfrac{\cos x(1+\cos x) - \sin x(-\sin x)}{(1+\cos x)^2}$	1	for correct differentiation
$= \dfrac{\cos x + \cos^2 x + \sin^2 x}{(1+\cos x)^2}$		
$= \dfrac{\cos x + 1}{(1+\cos x)^2}$		
$= \dfrac{1}{1+\cos x}$	1	for correct simplification
e) (i) $\dfrac{d}{dx}(x \ln x) = 1.\ln x + x.\dfrac{1}{x}$ $= \ln x + 1$	1	for correct differentiation
(ii) $\therefore \int (\ln x + 1)\,dx = x \ln x + c$	1	for correctly antidifferentiating
$\therefore \int \ln x\,dx = x \ln x - \int 1\,dx + c$ $= x \ln x - x + c$		
$= x(\ln x - 1) + c$	1	for correct integration

2U TRIAL 2015 **MATHEMATICS: Question** 11 JRAHS

Suggested Solutions	Marks	Marker's Comments
f) a, b, c in GP $\Rightarrow \dfrac{b}{a} = \dfrac{c}{b}$ $b^2 = ac$ Taking log: $\ln b^2 = \ln(ac)$ $\Rightarrow 2\ln b = \ln a + \ln c$ i.e $\ln b - \ln a = \ln c - \ln b$ Hence common difference, d exists for sequence $\ln a, \ln b, \ln c$ $\Rightarrow \ln a, \ln b, \ln c$ are in AP	1 1	for correct deduction from GP sequence for showing existence of common difference
g) (i) $x^2 - 10x - 16y - 7 = 0$ $16y = x^2 - 10x + 5^2 - 32$ $= (x-5)^2 - 32$ $\therefore (x-5)^2 = 16(y+2)$ $\therefore$ Vertex $(5, -2)$ (ii) focal length, $a = 4$ - - - - - - - - - - - - - - - - Method II: axis of symmetry, $x = \dfrac{-b}{2a}$ Method III: $y' = 0 \Rightarrow x = 5$ etc	 1 1 1	 for $x^2 = 4ay$ for correct vertex for focal length

2unit

MATHEMATICS: Question 12

Suggested Solutions	Marks	Marker's Comments

a) i) $T = T_0 e^{-kt}$

When $t = 0$, $T = 10000$

$\therefore T_0 = 10000$ — (1)

$\therefore T = 10000e^{-kt}$

When $t = 1$, $T = 9000$

$\therefore 9000 = 10000e^{-k}$

$\therefore e^{-k} = \frac{9}{10}$

$-k = \ln\left(\frac{9}{10}\right)$

$k = -\ln\left(\frac{9}{10}\right)$ or $\ln\left(\frac{10}{9}\right)$ — (1)

ii) $T = 40 \Rightarrow 40 = 10000e^{-\ln\left(\frac{10}{9}\right)t}$

$\therefore e^{-\ln\left(\frac{10}{9}\right)t} = 0.004$ — (1)

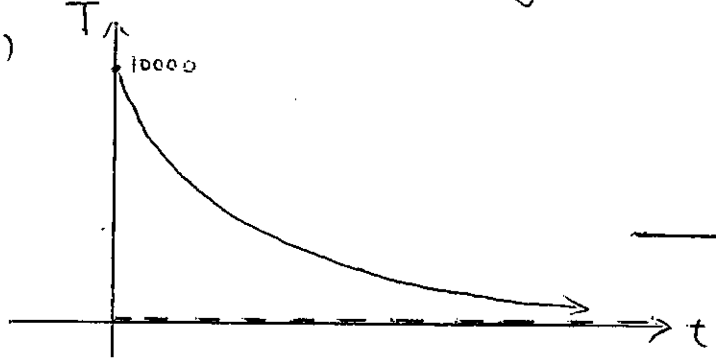
$\therefore -\ln\left(\frac{10}{9}\right)t = \ln(0.004)$

$t = \dfrac{-\ln(0.004)}{\ln\left(\frac{10}{9}\right)}$

$= 52.41$ (2dp) — (1)

$\therefore$ It'll be safe after 53 years

iii)

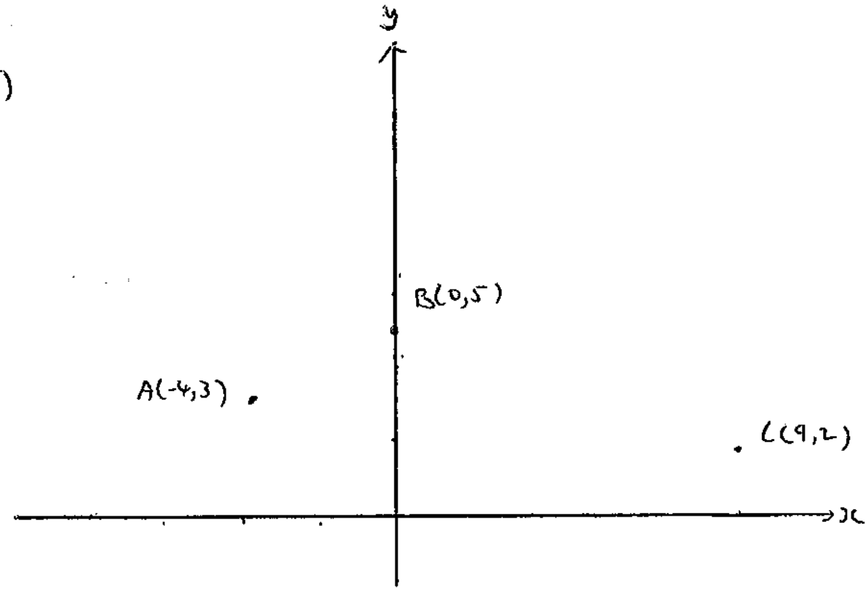


— (1)

Note: * $t \geq 0$

* Vertical Intercept

MATHEMATICS: Question…….

Suggested Solutions	Marks	Marker's Comments
b) i) 	1	Should really label A, B and C.
ii) $d(BC) = \sqrt{(9-0)^2 + (2-5)^2}$ $= \sqrt{81+9}$ $= \sqrt{90}$ $= 3\sqrt{10}$	1	
iii) $m_{BC} = \frac{5-2}{0-9} = \frac{-1}{3}$ $\therefore m_l = \frac{-1}{3}$ (parallel to BC)	1	
$\therefore l$ is given by $y-3 = \frac{-1}{3}(x+4)$ $3y-9 = -x-4$ $x+3y-5=0$	1	
iv) When $y=0$, $x+3(0)-5=0$ $\therefore x=5$. $\therefore D$ is given by $(5,0)$	1	Note: Must write as a coordinate!

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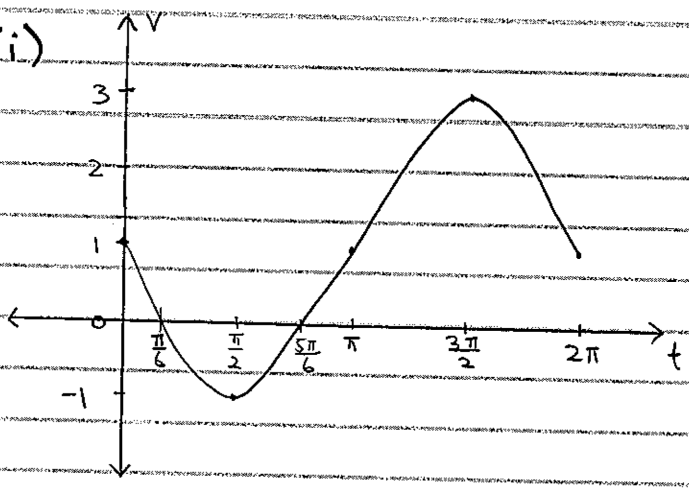
MATHEMATICS: Question……..

Suggested Solutions	Marks	Marker's Comments
v) $d_{AD} = \sqrt{(5-(-4))^2 + (0-3)^2}$		
$= \sqrt{81 + 9}$		
$= \sqrt{90}$		
$= 3\sqrt{10}$		
$= d_{BC}$	1	
Since $BC \parallel AD$ (given previously).		
ABCD is a parallelogram (a pair of opposite sides equal and parallel)	1	
vi) Perpendicular distance $= \dfrac{\lvert 1(0) + 3(5) - 5 \rvert}{\sqrt{1^2 + 3^2}}$	1	
$= \dfrac{10}{\sqrt{10}}$		
$= \sqrt{10}$ units	1	
vii) $A = 3\sqrt{10} \times \sqrt{10}$		
$= 30 \text{ units}^2$	1	

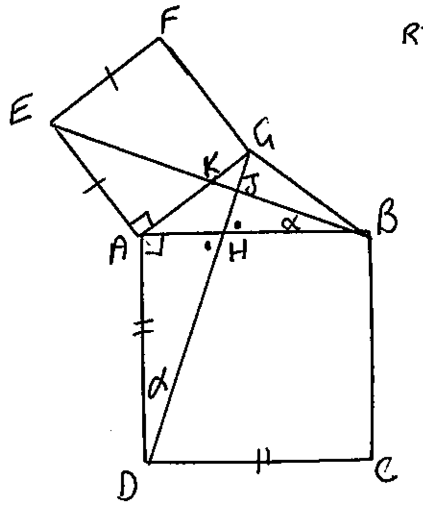
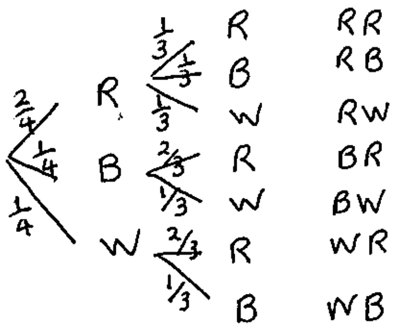
MATHEMATICS: Question 13

Suggested Solutions	Marks	Marker's Comments
a) i) $v = 1 - 2\sin t$		
$v = 0$, when particle is at rest		
$0 = 1 - 2\sin t$		
$2\sin t = 1$		
$\sin t = \frac{1}{2}$	1	
$t = \frac{\pi}{6}, \frac{5\pi}{6}$		
$\therefore t_1 = \frac{\pi}{6}$, $t_2 = \frac{5\pi}{6}$	1	
ii) 		1 shape 1 correct scale.
iii) $\ddot{x} = -2\cos t$	1	
when $t_1 = \frac{\pi}{6}$, $\ddot{x} = -2\cos\frac{\pi}{6}$ $= -\sqrt{3}\ m/s^2$		
$t_2 = \frac{5\pi}{6}$, $\ddot{x} = -2\cos\frac{5\pi}{6}$ $= \sqrt{3}\ m/s^2$	1	
iv) $v = 1 - 2\sin t$		
$\frac{dx}{dt} = 1 - 2\sin t$		
$\therefore x = t + 2\cos t + c$	1	
when $t = 0$, $x = 0$		
$0 = 0 + 2\cos(0) + c$		
$\therefore c = -2$	1	
$\therefore x = t + 2\cos t - 2$		

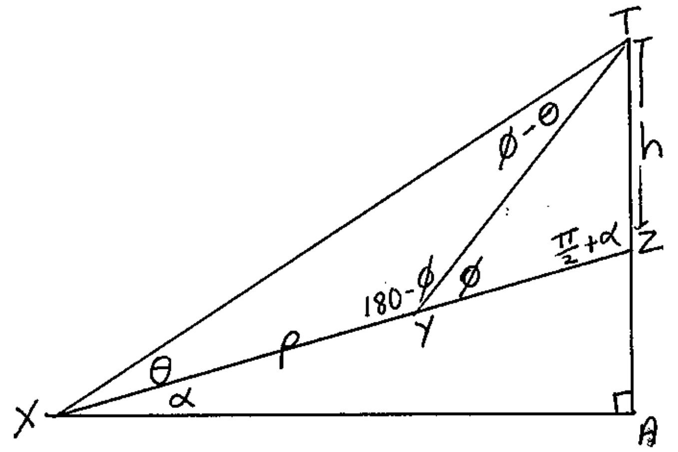
MATHEMATICS: Question 13

Suggested Solutions	Marks	Marker's Comments
v) when $t_1 = \frac{\pi}{6}$, $x_1 = \frac{\pi}{6} + 2\cos\frac{\pi}{6} - 2$		
$= \frac{\pi}{6} + \sqrt{3} - 2$		
$t_2 = \frac{5\pi}{6}$, $x_2 = \frac{5\pi}{6} - \sqrt{3} - 2$		
distance $= \left\lvert \frac{5\pi}{6} - \sqrt{3} - 2 - \left(\frac{\pi}{6} + \sqrt{3} - 2\right) \right\rvert$	1	many students did not include the absolute value for distance
$= \left\lvert \frac{4\pi}{6} - 2\sqrt{3} \right\rvert$		
$= 2\sqrt{3} - \frac{2\pi}{3}$	1	
b) sum of roots:		
$\alpha + \beta = \frac{(4k+1)}{2}$		
since $\alpha = -\beta$		
$-\beta + \beta = \frac{4k+1}{2}$	1	
$0 = \frac{4k+1}{2}$		
$4k+1=0$		
$4k = -1$		
$k = -\frac{1}{4}$	1	
c) $y = 5x - 7$		
$m = 5$		could also use:
$\therefore 4x - 3 = 5$		$5x-7 = 2x^2-3x+c$
$4x = 8$		$\therefore 2x^2-8x+k+7=0$
$x = 2$		$\Delta = 0$
when $x = 2$, $y = 5(2) - 7$		$\vdots$
$= 3$		$c = 1$
$\therefore$ the tangent cuts the curve at $(2,3)$	1	$\therefore y = 2x^2-3x+1$
$f(x) = 4x - 3$		
$F(x) = \int(4x-3)\,dx$		
$= 2x^2 - 3x + c$	1	
at $(2,3)$		
$3 = 2(2)^2 - 3(2) + c$		
$\therefore c = 1$		
$\therefore y = 2x^2 - 3x + 1$	1	

MATHEMATICS: Question 14

Suggested Solutions	Marks	Marker's Comments
RTP $\Delta ADG \equiv \Delta ABE$		
		Question generally very well done.
(i) In Δ's ADG and ABE • $AD = AB$ (given ABCD is a square). • $AG = AE$ (given AEFG is a square). $\angle DAB = \angle EAG = 90°$ (angles of a square). Now, $\angle EAB = 90 + \angle BAG$ and $\angle DAG = 90 + \angle BAG$ proven • $\therefore \angle DAG = \angle EAB$ $\therefore \Delta ADG \equiv \Delta ABE$ (SAS)	1mk 1mk	
(ii) In Δ's ADH + BHJ $\angle ADG = \angle EBA = \alpha$ (corresponding angles, $\Delta ADG \equiv \Delta ABE$) $\angle JHB = \angle AHD$ (vertically opposite angles) $\therefore \Delta ADH \parallel\!\mid \Delta BHJ$ (equiangular) $\therefore \angle DAH = \angle HJB = 90°$ (corresponding angles, similar triangles) $\therefore EB \perp DG$	1mk 1mk 1mk.	There were many other solutions using angle sums of triangles, quadrilaterals, exterior angles of triangles.
b) (i) 	1mk	for tree diagram
(ii) $P(RR) = \frac{2}{4} \times \frac{1}{3}$ $= \frac{1}{6}$	1mk	

MATHEMATICS: Question 14 page2.

Suggested Solutions	Marks	Marker's Comments
(iii) $P(\text{at least } 1 \text{ Red}) = 1 - P(\text{no red})$ $= 1 - [P(BW) + P(WB)]$ $= 1 - \left(\frac{1}{4} \times \frac{1}{3} + \frac{1}{4} \times \frac{1}{3}\right)$ $= 1 - \frac{1}{6}$ $= \frac{5}{6}$	1mk.	
(iv) Our sample space was RR, RB, RW, BR, BW, WR, WB. Since we know one of the balls is Red, we now only have 5 possible outcomes. $\therefore P(RR) = \frac{1}{5}$	1mk	
(c) 		
$\angle XTY + \theta + 180 - \phi = 180$ (angle sum ΔXTZ) $\angle XTY = \phi - \theta$	1mk	
Using the sin Rule $\frac{p}{\sin \angle XTY} = \frac{TY}{\sin\theta}$ $\frac{p}{\sin(\phi - \theta)} = \frac{TY}{\sin\theta}$ $TY = \frac{p\sin\theta}{\sin(\phi - \theta)}$	1mk.	this was answer given.

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MATHEMATICS: Question 14 page 3		
Suggested Solutions	**Marks**	**Marker's Comments**
(ii) $\angle YZT = \angle TAX + \angle AXZ$ (exterior angle of $\triangle AXZ$). $\therefore \angle YZT = \frac{\pi}{2} + \alpha$	1mk	(answer was given).
(iii) In $\triangle TYZ$ $$\frac{TY}{\sin \angle YZT} = \frac{h}{\sin\phi}$$ $$\frac{TY}{\sin\left(\frac{\pi}{2}+\alpha\right)} = \frac{h}{\sin\phi}$$	1mk	
Subst $TY = \frac{p\sin\theta}{\sin(\phi-\theta)}$ from (ii) $$\therefore \frac{p\sin\theta}{\sin(\phi-\theta)\sin\left(\frac{\pi}{2}+\alpha\right)} = \frac{h}{\sin\phi}$$ $$h = \frac{p\sin\theta\sin\phi}{\sin(\phi-\theta)\sin\left(\frac{\pi}{2}+\alpha\right)}$$	1mk	It was not sufficient to proceed from $\sin(\frac{\pi}{2}+\alpha)$ to $\cos\alpha$ unless some acknowledgement was given to them being equal.
Now: $\sin\left(\frac{\pi}{2}+\alpha\right) = \cos\left(\frac{\pi}{2}-\left(\frac{\pi}{2}+\alpha\right)\right) = \cos(-\alpha) = \cos\alpha.$ $$h = \frac{p\sin\theta\sin\phi}{\sin(\phi-\theta)\cos\alpha}$$	1mk.	this was answer given

MATHEMATICS: Question 15

Suggested Solutions	Marks	Marker's Comments

a) $4x \leq 15 \leq -9x$

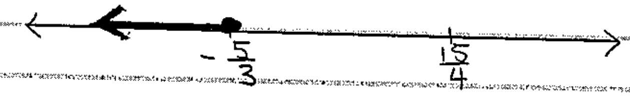
$4x \leq 15$ or $15 \leq -9x$

$x \leq \dfrac{15}{4}$ $\qquad$ $-9x \geq 15$

$\qquad\qquad x \leq \dfrac{-15}{9}$

$x \leq 3\dfrac{3}{4}$ $\qquad$ $x \leq -\dfrac{5}{3}$ — 1

$\therefore x \leq -\dfrac{5}{3}$ — 1



1

b) i) $A = \int_0^{\frac{\pi}{6}} \tan 2x \, dx$

$= \int_0^{\frac{\pi}{6}} \dfrac{\sin 2x}{\cos 2x} \, dx$ — 1

$= -\frac{1}{2}\left[\ln(\cos 2x)\right]_0^{\frac{\pi}{6}}$ — 1

$= -\frac{1}{2}\left(\ln(\cos \frac{\pi}{6}) - \ln(1)\right)$

$= -\frac{1}{2} \ln \frac{1}{2}$

$= \frac{1}{2} \ln 2$ — 1

ii) $V = \pi \int_0^{\frac{\pi}{6}} \tan^2 2x \, dx$

x	0	$\frac{\pi}{24}$	$\frac{\pi}{12}$	$\frac{\pi}{8}$	$\frac{\pi}{6}$
$\tan^2 2x$	0	0.072	$\frac{1}{3}$	1	3

1

$V = \pi\left(\dfrac{\pi}{12\times 6}\left(0 + 4\left(\tan^2\left(\tfrac{\pi}{12}\right)\right) + \tfrac{1}{3}\right)\right.$ — 1

$\left. + \dfrac{\pi}{12\times 6}\left(\tfrac{1}{3} + 4\times 1 + 3\right)\right)$

$= 1.0902970\ldots$ (calc. display)

$= 1.0903$ (4 dp) unit^3 — 1

Marker's comments: need to show that 2 applications were used or 5 function values

MATHEMATICS: Question 15

Suggested Solutions	Marks	Marker's Comments
c) i) $y = \frac{4}{x}$		
$y' = -\frac{4}{x^2}$		
at $P(2t, \frac{2}{t})$		
$y' = \frac{-4}{(2t)^2}$		
$= -\frac{1}{t^2}$	1	
$\therefore$ equation of tangent at P is:		
$y - \frac{2}{t} = -\frac{1}{t^2}(x - 2t)$		need to show all steps!
$t^2y - 2t = -x + 2t$	1	
$t^2y = -x + 4t$		
$t^2y = 4t - x$		
ii) $t^2y = 4t - x$		
when $x = 0$, $y = \frac{4}{t}$		
$\therefore$ A is $(0, \frac{4}{t})$		
when $y = 0$, $x = 4t$		
$\therefore$ B is $(4t, 0)$		
$V = AB^2$		NOTE: $V = AB^2$ <u>NOT</u> $V = AB$
$= (4t)^2 + \left(\frac{4}{t}\right)^2$		
$= 16t^2 + \frac{16}{t^2}$	1	
$V' = 32t - \frac{32}{t^3}$	1	
for stationary points $V' = 0$		
$0 = 32t - \frac{32}{t^3}$		
$0 = 32t^4 - 32$		
$t^4 = 1$		

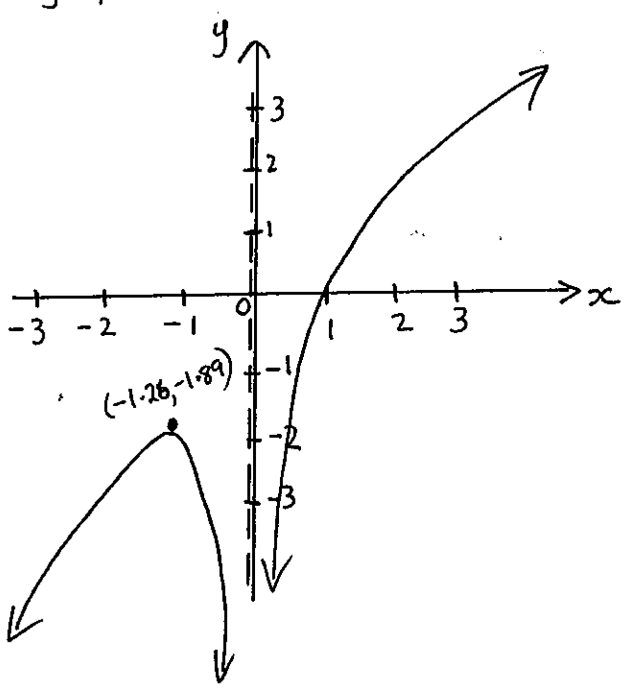
MATHEMATICS: Question 15

Suggested Solutions	Marks	Marker's Comments
$\therefore t = \pm 1$ but t is in the first quadrant $\therefore t = 1$	1	need to state why it is the positive one.
$v'' = 32 + \frac{96}{t^4}$ at $t=1$ $v'' = 32 + 96 > 0$ $\therefore$ the curve concaves up at $x=1$ $\therefore t=1$ is a minimum	1	can also use the table for the gradient

x	0.9	1	1.1
v'	-15	0	11
	\	—	/

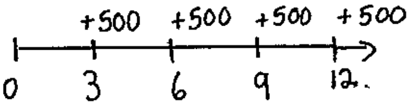
MATHEMATICS: Question 16 Page 1.

Suggested Solutions	Marks	Marker's Comments
(a) $\log_5 8 = a$ $\log_5 2^3 = a$ $a = 3\log_5 2$ $a = \dfrac{3\log_{10}2}{\log_{10}5}$ $a = \dfrac{3\log_{10}2}{\log_{10}\left(\frac{10}{2}\right)}$ $a = \dfrac{3\log_{10}2}{\log_{10}10 - \log_{10}2}$ $a = \dfrac{3\log_{10}2}{1-\log_{10}2}$ $a(1-\log_{10}2) = 3\log_{10}2$ $a - a\log_{10}2 = 3\log_{10}2$ $3\log_{10}2 + a\log_{10}2 = a$ $\log_{10}2\,(3+a) = a$ $\log_{10}2 = \dfrac{a}{3+a}$		Note: Do not use $\log 2$ for $\log_{10}2$. $\log 2 = \log_e 2$ $= \ln 2$.
Alternatively: If $\log_5 8 = a$ $3\log_5 2 = a$ $\log_5 2 = \dfrac{a}{3}$		
$\therefore \log_{10}2 = \dfrac{\log_5 2}{\log_5 10}$	1mk	
$= \dfrac{\frac{a}{3}}{\log_5(2\times5)}$ $= \dfrac{\frac{a}{3}}{\log_5 2 + \log_5 5}$	1mk	
$= \dfrac{\frac{a}{3}}{\frac{a}{3}+1}$ $= \dfrac{\frac{a}{3}}{\frac{a+3}{3}}$	1mk.	
$\log_{10}2 = \dfrac{a}{a+3}$		
(b) (i) $f(x) = x - x^{-2}$ $f'(x) = 1 + 2x^{-3}$ $= 1 + \dfrac{2}{x^3}$ $f''(x) = -6x^{-4}$ $= \dfrac{-6}{x^4}$	1mk	
Since $x^4 > 0$ then $\dfrac{-6}{x^4} < 0$ for all x Since $f''(x) < 0$ the graph of $f(x)$ is always concave down.	1mk	

MATHEMATICS: Question 16 page 2		
Suggested Solutions	**Marks**	**Marker's Comments**
(ii) When $f(x) = 0$ $\quad x - \frac{1}{x^2} = 0 \quad x \neq 0$ $x^3 - 1 = 0$ $x = 1$ $\therefore f(x)$ cuts the x axis at $x=1$ ie $(1,0)$. Stationary points occur when $f'(x)=0$ ie when $1 + \frac{2}{x^3} = 0$ $x^3 + 2 = 0$ $x = -\sqrt[3]{2}$ $x = -1{\cdot}26$ When $x = -1{\cdot}26$ $y = -1{\cdot}8899$. Since $f(x)$ is concave down for all x, then $(-1{\cdot}26, -1{\cdot}89)$ is a maximum turning point.  		1 mk for cutting axis at $x=1$ and having an asymptote at $x=0$. 1 mk for finding the stationary point. 1 mk for having the concave down and correct shape for -ve x values.

Suggested Solutions	Marks	Marker's Comments

(c)



$r = 8\%$ pa

$= 2\%$ for 3 mths

(i) 1st \$500 is invested for 3 terms & amounts to $500(1{\cdot}02)^3$ — 1mk

2nd \$500 is invested for 2 terms & amounts to $500(1{\cdot}02)^2$

3rd \$500 " " " 1 term " " " $500(1{\cdot}02)^1$

4th \$500 " " " 0 terms " " " $500(1{\cdot}02)^0$

$\therefore$ Total amount after 1st birthday

$$S_4 = 500(1{\cdot}02)^0 + 500(1{\cdot}02)^1 + 500(1{\cdot}02)^2 + 500(1{\cdot}02)^3$$

$$= 500\left(1 + (1{\cdot}02)^1 + (1{\cdot}02)^2 + (1{\cdot}02)^3\right)$$

$$= \$\,2060{\cdot}80$$

1mk

(ii) 15 years = 60 terms

$$S_{60} = 500\left(1 + (1{\cdot}02)^1 + (1{\cdot}02)^2 + (1{\cdot}02)^3 + \ldots + (1{\cdot}02)^{59}\right)$$

1mk

$a = 500$

$r = 1{\cdot}02$

$$S_{60} = \frac{500\left(1{\cdot}02^{60} - 1\right)}{1{\cdot}02 - 1} \qquad \text{ie } S_n = \frac{a(r^n - 1)}{r - 1}$$

1mk

$$= \$\,57025{\cdot}77$$

(iii) No more deposits are made in the 16th year, but interest is added to \$57025·77.

$$\therefore A_{16} = \$\,57025{\cdot}77\,(1{\cdot}02)^4$$

$= \$\,61726{\cdot}53$ is in accout after 16th birthday

1mk

(iv) We can see from (iii) that Robbie earned $(61726{\cdot}53 - 57025{\cdot}77)$ interest.

ie \$4700·76 interest.

If Robbie were to withdraw \$4000 each year his account is still accruing money because he is earning \$4700 interest.

If he withdrew \$5000 each year, he is taking out more than the interest earned. His account balance would decrease.

Note: \$5000 will empty accout out faster = 0 mks.

- lose money more quickly with \$5000 = 0 mks

1mk

1mk